# GEOMETRICAL-PARAMETER OPTIMIZATION FOR A PLANAR-SECTIONAL HEAT-EXCHANGER TUBE

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The proper choice of feature to be maximized or minimized is very important in optimizing heat exchangers; the relevant quantity may be the maximum heat flux, the reliability, the available reserve handling capacity, the temperature difference, the cost, or the weight. In the ideal case, all these factors have to be weighed, with an appropriate importance assigned to each. In practice, it is impossible to consider such a global optimum, and even with considerable simplifications the problem is not simply that of finding the turning point in a function of one variable. In most published papers on optimization, a single purpose is discussed, namely, maximizing the heat flux density in the cross section. In many cases of practical importance, this is the wrong quantity to choose, and an optimal heat exchanger should have minimal mass. Of course, the detailed parameters that govern the mass are peculiar to the design, so there is no general solution, although certain general trends can be detected in any particular case.

The mass of the device shown in Fig. 1 is a function of five variables:  $G = G(h, l_k, n, s, \delta)$ , where n is the number of pipes; of these five variables, two (h and  $l_k$ ) are dependent, while the other three are independent. The relation between them may be derived from the hydrodynamic heat-transport equation for the pipe in conjunction with the heat-balance equation. These are restrictive equations, and the other restrictions are as follows: the limiting heat flux density in the evaporator, the maximum and minimum wall thicknesses, and the number of pipes in the complete cross section, all of which are expressed as inequalities. The problem may be solved numerically. The minimum mass lies in the range adjoining the lower boundary for the wall thickness, so the search in  $\delta$  must be begun there. The pipe



Fig. 1. Design of heat-exchanger tube.

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thicknesses and number of pipes may be varied fairly widely, but the product, i.e., the total width of the wall, should not exceed 20% of the width of the entire section. Calculations on particular examples show that in most cases the minimum mass is obtained for the minimum possible wall thickness, and this is in noway related to attainment of the maximal axial heat flux density.

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HEAT TRANSFER THROUGH THE BASE OF A SET OF RECTANGULAR FINS IN COOLING HUMID AIR

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Heat and mass transfer in the cooling of moist air may be accelerated in compact heat exchangers employing rectangular metal fins; the variable parameters of the air flowing between the fins may be derived from the effective heat-transfer coefficient for the air with respect to the base of the fins, this quantity being dependent on the temperature and relative humidity.

The temperature gradient in a fin at the base is determined subject to the following restrictions:

1) the temperature, water-vapor content, air speed, fin temperature, and base temperature are constant in the cross section of the exchanger;

2) the coefficient  $\alpha_0$  for convective heat transfer in the absence of condensation is constant over the height H of a fin, which is true for moderate heights in the range H  $\sqrt{2}_0/\Delta\lambda_r \leqslant 2$ , with the upper limit corresponding to the maximum heat transfer through the base of a fin of thickness  $\Delta$ ;

droplet condensation occurs;

4) the condensate does not accumulate in the spaces between the fins;

5) the thermal conductivity  $\lambda_f$  of the fin material is independent of temperature, as are the specific heats of the mixture and the latent heats of condensation.

The height of a fin is divided into two parts. In the first part, which lies near the base, the temperature at the surface of the fin is less than the saturation temperature of the air, whereas in the upper part the temperature is above the saturation point. Condensation occurs in the first part. The heat-transfer factor  $\alpha$ for the air on the first part is derived from the relation for the heat and mass transfer for droplet condensation as a function of the temperature difference between the air and the wall, the vapor content of the air, the vapor content of saturated air at the wall temperature, and the heat-transfer coefficient  $\alpha_0$  for the second part. Further, differential equations are solved for the one-dimensional thermal conduction in a fin for the first and second parts with an exponential approximation for the water-vapor content of saturated air as a function of temperature. The solutions have been derived as the temperature dependence of the temperature gradient in the fin in the first and second parts, using boundary conditions at the end of the fin (temperature of the thermally insulated end specified) and at the boundary between the parts (equality in the fin temperatures and temperature gradients). It is shown that the temperature of the end may be taken as zero in the case of the heat-transfer coefficient at the base in relation to the relative humidity and air temperature, as well as the temperature of the base, the geometry of the fins, and  $\alpha_0$ . The ratio of fin thickness to width of gap between fins has been optimized to give maximum heat transfer. A simplified expression is given for the heat flux density at the base of a fin for saturated air.

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We study an isotropic plate whose internal and external contours are given by the equations

 $\begin{aligned} x &= \mu \cos \vartheta, \quad x = \cos \vartheta + \varepsilon \cos n\vartheta, \\ y &= \mu \sin \vartheta; \quad y = \sin \vartheta - \varepsilon \sin n\vartheta, \end{aligned}$ 

where  $\mu = R_1/R_2 < 1$ ;  $R_1$ ,  $R_2$ ,  $\varepsilon$ , n are constants;  $|\varepsilon| < 1$ ;  $\vartheta$  is the parameter determining the position of the points of the contour.

The heat exchange with the medium is realized through the bases of the plate according to Newton's law. The temperature of the medium varies in time according to an exponential law and is equal to zero at the initial moment. In the following moments the temperature does not vary on the internal lateral face of the plate, and the temperature of the external lateral face is a time function. We assume that the heat characteristics of the material do not depend on the temperature.

The temperature function is sought in the form of powers of the small parameter. The quantity  $\varepsilon$  is assumed for the parameter. The boundary conditions on the external noncircular contour are transformed such that the desired function contained in them is expanded in Taylor series. The representation in Taylor series enables us to express the function on the complex curvilinear contour in terms of the values of the functions and its particular derivatives on the circular contour. After substituting the desired temperature function into the basic differential equation of the problem and the transformed boundary conditions, and then comparing the coefficients for identical powers of  $\varepsilon$ , we obtain a recurrent sequence of boundary-value problems for the annular region.

The solution of the nonsteady problems of heat conduction for the annular region is found by means of a Laplace integral transformation [1]. The transfer from the transforms to the originals is carried out by means of an expansion theorem [1]. The law of temperature distribution in the plate under study is established with accuracy up to the fourth approximation.

We study an example of calculating the temperatures in a square plate with the internal circular contour (n = 3,  $\varepsilon = -1/9$ ). The temperature distribution is examined at various times. The results of the calculations are represented in a table and a graph. We estimate the accuracy of the solution obtained by the degree to which the boundary conditions on the curvilinear contour are satisfied.

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STUDY OF THE HEATING OF MATERIALS AND THE EVAPORATION OF SOLVENTS IN A MEDIUM OF SUPERHEATED STEAM

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We carry out an analysis of the solution of problems with a moving boundary under the assumption of a quasistationary temperature distribution: the absences of a temperature gradient in the material and of a linear temperature variation along the thickness of the condensation film. As a result we show and verify by experiments on plates made from Teflon, pine, and technical fibers that a quasistationary model is unsuitable for heating poor heat conductors in a medium of superheated steam. Solutions of a heat equation are recommended for approximate calculations of the heating of such materials under boundary conditions of the first kind  $T(R, \tau) = T_{bp}$ . However, the calculation under these conditions nevertheless yields a film thickness 5-15% higher in comparison with the experiment at the moment when the condensation  $\tau_{bp}$  ends, and the measured temperatures of the surface  $T(R, \tau_{bp})$  are 3-8°C lower than  $T_{bp}$ . Thus we conclude that the temperature of the free surface of the liquid under the nonstationary conditions of a rapidly growing condensation film is lower than the saturation temperature. Empirical linear and exponential approximations for the increasing temperature of the surface of the materials in time are suggested in order to refine the calculations.

Exponents for the evaporation of benzine, toluene, and xylene that saturate technical fibers made from anide and Kapron showed the presence of two areas of temporary temperature stabilization in the material - first for the temperature of the azeotrope  $T_{az}$ , and then for the temperature of boiling water  $T_{bp} > T_{az}$ . An approximate formulation of the problem is given for the evaporation of organic solvents in a medium of superheated steam, and the solutions of the problems are obtained which show satisfactory agreement with the experiment.

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### ANALYSIS OF GAS PRESSURE IN POROUS CERAMICS

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In [1, 2] it is shown on the basis of an analysis of the experimental data for heat conduction in ceramic materials for the evacuation of a gas medium that the pressure on the boundaries between the grains (in thin semiclosed pores) can exceed the pressure in a vacuum chamber by several orders. This effect is explained by the presence of a sufficient number of gas sources in the ceramic material, in particular, by the expansion of certain mixtures and the large hydraulic resistance of micropores on the boundary between the grains. We call the gas formed in this case the "active" gas in contrast to the "passive" gas filling the chambers and pores before the evacuation begins. One of the arguments, on the basis of which the above hypothesis is confirmed in [1], is the estimation of the energy of activation in the degassing process in the ceramic material on the basis of a Clapeyron-Clausius equation. It is not altogether clear to what degree this equation is applicable for the analysis of the nonequilibrium degassing process.

In the present study we additionally analyze the possibility of applying the Clapeyron-Clausius equation to the case under study on the basis of estimating the divergence of the pressure of the "active" gas in the pores from the equilibrium pressure.

We study the process when the outflow of "active" gas from the surface of the ceramic material, in view of its evaporation or desorption, occurs in a medium with the gas temperature and pressure  $T_n$  and  $P_n$ , which are distinguished from the equilibrium temperature and pressure  $T_e$  and  $P_e$ . If we guarantee the continuous discharge of the "active" gas from the surface of the material by means of an external action (a vacuum pump, for example) so that the (nonequilibrium) pressure  $P_n$  is lower than the equilibrium pressure  $P_e$ , the result will be a gas flow directed away from the surface with a certain velocity  $U_n$ . We can show that the mass flow  $T_n$  that is directed away from a unit of the surface is equal to

$$m_{\rm n} = \frac{P_{\rm e}}{\sqrt{\frac{2\pi RT_{\rm e}}{\mu}}} - \frac{P_{\rm n}}{\sqrt{\frac{2\pi RT_{\rm n}}{\mu}}} \sqrt{\pi} \, \text{ierfc} \left(\frac{u_{\rm n}}{\sqrt{\frac{2\pi T_{\rm n}}{\mu}}}\right); \tag{1}$$

here  $\mu$  is the molecular weight of the gas, and R is a universal gas constant.

If the quantity  $m_n$  is known, then from Eq. (1) we can determine the parameters of the nonequilibrium "active" gas in the volume over the surface.

The calculations carried out on the basis of (1) by specially transformed and approximated simple functions show that for the cases studied in [1, 2] the gas pressure between the grains of the ceramic material can be considered as equilibrium pressure with an error of not more than 0.1%, and the calculation of the energy of activation in the degassing process is valid on the basis of the data for the pressure  $P_n$  in the micropores.

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## TEMPERATURE DISTRIBUTION IN A MULTILAYER BODY AT SMALL TIMES

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A one-dimensional equation with the nonuniformity a function of time can be used to describe the temperature distributions in planar, cylindrical, and spherical walls; boundary conditions of the third kind apply at the outer boundaries, with the nonuniformity a function of time. Ideal contact occurs between the layers. A solution has been given previously to this problem [1] that applies for large times; for small times the representation is

$$\frac{\delta_{qi}}{\Delta} = \frac{\varepsilon_{qi}}{C} \cdot \frac{1}{1 - \frac{1}{C} F(s)}, \qquad (9)$$

where  $\triangle$  is the determinant of the inhomogeneous system of boundary and contact equations from [1],  $\delta_{qi}$  is a partial determinant, C is a constant, and F(s) is a Laplace transform. A scheme is given for calculating  $\triangle$  of order 2k, where k is the number of layers. The left side of (9) is equated to the right by expressing the hyperbolic functions in terms of exponentials, with the cylindrical functions replaced by an asymptotic expansion. The second fraction in the right in (9) is expanded as a power series, which enables one to invert the series terms for the temperature in terms of the function

$$(2\sqrt{\tau})^m i^m \operatorname{erfc} \frac{z}{2\sqrt{\tau}}$$
.

If all the terms in the asymptotic expansion apart from the first are discarded, then the limiting values for the hyperbolic functions at infinity are employed, one derives simple tabulated functions, whose originals give approximate values for the temperature suitable for small times. The argument of the hyperbolic functions is zero at certain boundaries. The general formula for passing to the limit is inapplicable and results in considerable error in the temperature. A published method [2] has been used to refine the result, and the system of boundary values for the heat-transfer factors is considered separately. A table is given of results for planar, cylindrical, and spherical three-layer walls. Three solutions are compared: exact, approximate, and by one-dimensional approximation.

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A. V. Lykov, Theory of Thermal Conduction [in Russian], GITTL, Moscow (1952).

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EFFECTS OF VARIABLE RHEOLOGICAL PARAMETERS ON HEAT TRANSFER IN LAMINAR FLOW OF A NON-NEWTONIAN LIQUID IN A TUBE

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Many measurements show that one can incorporate the effects of variable viscosity on heat transfer to Newtonian liquids in pipes and in other cases of laminar flow (for instance, in mixers) in terms of a power-law correction containing the viscosities at the temperatures of the liquid and the wall,  $(\mu_b/\mu_W)^{0.14}$ ; however, the physical meaning of these corrections and the limits of application have not been clearly established; standard methods of extending the corrections to non-Newtonian systems have not been shown to have any theoretical basis, and the results are in disagreement with experiment.

In the case of viscous liquids (Pr  $\gg$  1), and at the flow speeds usually realized in practice (Pe > 10<sup>4</sup>), the rate of laminar heat transfer is dependent mainly on the heat transport near the wall, whereas the temperature distribution and heat transport at the core of the flow play only minor parts. For this reason, the gradient in the axial velocity at the wall  $\gamma_W$  remains decisive even when the viscosity is variable. Theoretical results have been processed to give the following relationship for the ratio of the Nu for variable and constant rheological parameters:

$$\frac{Nu^{nis}}{Nu^{is}} = \frac{\overline{Nu}^{nis}}{Nu^{is}} = \left(\frac{\dot{\gamma}_{w}^{nis}}{\dot{\gamma}_{w}^{is}}\right)^{S} = \sigma_{w}^{S}, \qquad (1)$$

where S = 0.23 for cooling or 0.29 for heating.

From (1) one gets that v = 0.14; the relationship indicates that the apparent shear viscosity at the wall  $\mu_W^a$  should be used as the analog of the viscosity for a non-Newtonian fluid; if the viscosity is introduced as the denominator in the generalized Reynolds number  $\mu^*$ , the value of v in the power-law correction should be dependent on the rheological parameters of the liquid (Fig. 1). For a liquid of power-law type, v = 0.14 m; for a nonlinearly viscoplastic liquid,

$$v = 0.14m + (0.46 + 0.38 \lg m)a, \tag{2}$$

where m = 1/n and  $\alpha$  is the relative radius of the rod section of the flow. Equation (2) fits closely to the measurements.



Fig. 1

The value of the parameter 0.14 reflects the deviation of the actual  $\dot{\gamma}_W$  from  $\dot{\gamma}_W, W$ ;  $\dot{\gamma}_W \rightarrow \dot{\gamma}_W, W$  for small values of the reduced length, and hence v for m = 1 should approach S (the curve of slope S is shown by a broken line in Fig. 1).

Power-law corrections apply when the power-law relationship  $\sigma_W = \mu_{W,b}^a/\mu_{W,W}^a$ still applies; in the case of tubes, the region is restricted to very small reduced lengths (l/Pe • x/d < 5 • 10<sup>-4</sup>), which are virtually unrealized in heat exchangers. Dep. 1238-75, March 3, 1975. All-Union Scientific-Research Institute of the Oil-Refining and Petrochemical Industry. Original article submitted February 18, 1974.

BOUNDARY EFFECTS AND THE FORMATION OF FREE SPACE IN AN INSULATING LIQUID

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UDC 537.84

The formation of free space charge in a liquid is a complex phenomenon that has not been extensively examined; difficulties arise in explaining some electrohydrodynamic effects on account of lack of knowledge on the causes of free space charge. Also, there are no direct methods of measuring space charge, and the extent and character of the charge have to be judged from the potential, ionic mobility, and electrohydrodynamic results. A survey of studies on this topic is presented.

Data on static electrification indicate that the latter is dependent on the metal, the water content in the liquid, and any traces of impurity of electrolyte type. The electrochemical and hydrodynamic effects of the charge formation are complex, and this prevents one from predicting the exact magnitude of the charge.

Current measurements indicate that there is a relationship between the charge production during electrification and in an external electric field; the more active the metal, the more its ions tend to enter the surrounding medium and the greater the current and electrification.

If the space-charge density is represented as  $\rho_{el} = \sum_{\alpha} e_{\alpha} + \frac{1}{\alpha} - \sum_{\alpha} e_{\alpha} - \frac{1}{\alpha}$ , the con-

servation of charge goes with Ohm's law to indicate that the charged transport in a real insulator is defined by the following system of equations:

 $\begin{aligned} \frac{\partial \rho_{\pm 1}^{+}}{\partial t} + \operatorname{div} \mathbf{j}^{+} &= k - \alpha n^{+} n^{-}, \\ \mathbf{j} &= \mathbf{E} \left( e z^{+} n^{+} b^{+} + e z^{-} n^{-} b^{-} \right), \\ \operatorname{div} \mathbf{E} &= \frac{\rho_{e1}}{\epsilon_{0} \epsilon}, \ \mathbf{E} &= -\nabla \varphi. \end{aligned}$ 

Here t is time, j is current density, k is degree of dissociation,  $\alpha$  is the recombination constant, n is the volume concentration of positive and negative ions, z is the valency, e is the electronic charge, b is the ionic mobility, E is the electric field strength,  $\varepsilon_0$  is a dielectric constant,  $\varepsilon$  is a dielectric constant, and  $\varphi$  is the electrical potential.

Measurements on the potential distribution indicate that the field strength varies considerably over the cross section between plane-parallel electrodes, and this is responsible for the free space charge; the current increases with the water content, but the bulk concentration of uncompensated charge decreases.

The ionic mobilities are dependent on the external electric field; there is also a relationship between the viscosity and ionic mobility.

The free-charge formation is thus of electrochemical nature, and liquid insulators have to be considered as dilute electrolytes on the basis that the carrier concentration is small, while the concentration of uncompensated charge is high.

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